# Computer adaptive testing for small scale programs and instructional systems

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This study investigates measurement decision theory (MDT) as an underlying model for computer adaptive testing when the goal is to classify examinees into one of a finite number of groups. The first analysis compares MDT with a popular item response theory model and finds little difference in terms of the percentage of correct classifications. The second analysis examines the number of examinees needed to calibrate MDT item parameters and finds accurate classifications even with calibration sample sizes as small as 100 examinees.

Computerized adaptive testing offers numerous advantages to both the test sponsor and the test taker. Test questions can be selected based on interim ability estimates and answers to previous test questions. The result is a tailored test able to maintain desired precision levels with fewer questions.

Much of the research on adaptive testing has centered on the use of item response theory (IRT) as the underlying model. While very attractive and widely used, IRT is fairly complex, relies on several restrictive assumptions, and typically needs large numbers of examinees to obtain satisfactory item parameter estimates.

This article presents an adaptive testing procedure using measurement decision theory (MDT) and compares the procedure with an IRT procedure in terms of classification accuracy. Accuracy is evaluated as a function of model, pool size, and calibration size.

## Overview

In the introduction to their classic textbook, Cronbach and Gleser (1957) argue that the ultimate purpose for testing is to arrive at qualitative classification decisions. When the goal is to classify examinees into one of a finite number of categories, such as certify/deny certification or advanced/proficient/basic/below basic, IRT is traditionally the model of choice. IRT provides a point estimate on a continuum as the indicator of ability of each examinee, and classification is finessed by identifying the category that encompasses the point estimate. In terms of adaptive mastery testing, the field has not advanced much beyond the early studies of Kingsbury and Weiss (1979, 1981, and 1983). For a good overview of the literature on computerized classification testing, see Thompson (2007).

Decision theory provides an alternative underlying model. Key articles in the mastery testing literature of the 1970s employed decision theory (Hambleton & Novick, 1973; Huynh, 1976; van der Linden & Mellenbergh, 1977). Lewis and Sheehan (1990), Kingsbury and Weiss (1983), Reckase (1983), and Spray and Reckase (1996) have used decision theory to adaptively select items and testlets and determine when to stop testing. Notable articles by Macready and Dayton

(1992), Vos (1999), Welch and Frick (1993), and Rudner (2002, 2009) illustrate the less prevalent item-level application of decision theory examined in this paper.

In this overview, MDT is described along with the investigated approach to adaptively administer test questions. A more detailed description can be found in Rudner (2009).

**Measurement Decision Theory**

The objective of MDT is to form a best estimate of the true classification of an individual examinee based on the examinee's item responses, *a priori* item information, and possibly *a priori* population classification proportions. Thus, the decision theory model has four components: 1) possible mastery states for an examinee, 2) calibrated items, 3) an individual's response pattern, and 4) decisions that may be formed about the examinee.

There are $K$ possible classifications that take on values $m_k$. In the case of pass/fail testing, there are two possible classifications and $K = 2$, with $m_1$ = "pass" and $m_2$ = "fail." The second component is a set of items for which the probability of each possible observation, usually right or wrong, is given for each classification. Just as with IRT CAT, these conditional probabilities of right or wrong answers are computed *a priori*. These conditional population probabilities can be determined through a variety of ways, including prior testing, transformations of existing scores, existing classifications, and possibly judgment. One approach to computing the conditional probabilities is to compute the proportions of examinees that respond correctly from a small pilot test involving examinees that have already been classified.

The responses to a set of $N$ items form the third component. Each item is considered to be a discrete random variable stochastically related to the classification levels and realized by observed values $z_N$. Each examinee has a response vector, **z**, composed of $z_1, z_2, ..., z_N$. Only dichotomously scored items are considered in this paper.

The last component is the decision space. One can form any number of $D$ decisions based on the data. Typically, one wants to make a classification decision and there will be $D = K$ decisions. With adaptive testing, a decision to continue testing can be added and thus there would be $D = K + 1$ decisions.

This paper uses the following notation:

Priors
$P(m_k)$ — the probability of a randomly selected examinee having mastery state $m_k$. In the absence of information and when classification priors are deemed inappropriate, uniform priors can be used.

$P(z_i/m_k)$ — the probability of response $z_i$ given the $k$-th mastery state

Observations
**z** — an individual's response vector $z_1, z_2, ..., z_N$, where $z_i \in (0,1)$

An estimate of an examinee's true classification is formed using the priors and observations. By Bayes Theorem,

$$P(m_k \mid z) \; = \; c \;\; P(z \mid m_k) \; P(m_k) \tag{1}$$

The posterior probability $P(m_k/z)$ that the examinee is of mastery state $m_k$ given his response vector is equal to the product of a normalizing constant ($c$), the probability of the response vector given $m_k$, and the prior classification probability. For each examinee, there are $K$ probabilities, one for each mastery state. The normalizing constant in formula (1),

$$c = \frac{1}{\displaystyle\sum_{k=1}^{K} P(z \mid m_k) \; P(m_k)}$$

assures that the sum of the posterior probabilities equals 1.0.

Assuming local independence,

$$P(z \mid m_k) \; = \prod_{i=1}^{N} P(z_i \mid m_k) \tag{2}$$

That is, the conditional probability of the response vector is equal to the product of the conditional probabilities of the item responses. In decision theory, the local independence assumption is also called the "naive Bayes" assumption. We will naively assume the assumption is true and proceed with our analysis.

The decision is usually made to classify the individual into the most likely group. If non-uniform priors are used, the approach is called the *maximum a posteriori* (MAP) decision criterion. If the priors are uniform, this is simply the maximum likelihood decision criterion. MAP itself is a subset of a third more general approach called the Bayes risk criterion, which incorporates weights for false negative and false positives.

**Measurement Decision Theory Adaptive Testing Using Information Gain**

Rudner (2002, 2009) describes three approaches to adaptive item selection using decision theory —information gain, minimum expected cost, and maximum discrimination. In that analysis, there was little difference in the effectiveness of information gain and minimum expected cost, both performing better than maximum discrimination. In this paper, only Information Gain was applied.

The concept of Information Gain builds on the widely used measure of information from information theory, Shannon entropy (Shannon, 1948; see Cover & Thomas, 1991):

$$H(S) = \sum_{k=1}^{K} -p_k \log_2 p_k \tag{3}$$

where $p_k$ is the proportion of $S$ belonging to class $k$. Base 2 is used for the logarithm because the formula was originally developed for binary data. Entropy can be viewed as a measure of the uniformity of a distribution and has a maximum value when $p_k = 1/K$ for all $k$. Since the goal is to have a peaked distribution of $P(m_k)$, one wants the lowest possible value of $H(S)$. Thus, one should next select the item that has the greatest expected reduction in entropy, i.e. $H(S_0) - H(S_i)$, where $H(S_0)$ is the current entropy and $H(S_i)$ is the expected entropy after administering item $i$. This expected entropy is the sum of the weighted conditional entropies of the classification probabilities that correspond to a correct and an incorrect response:

$$H(S_i) = p(z_i = 1) \, H(S_i | z_i = 1) + p(z_i = 0) \, H(S_i | z_i = 0) \tag{4}$$

This can be computed using the following steps:

1. Compute the normalized posterior classification probabilities that result from a correct and an incorrect response to item $i$ using (1).
2. Compute the conditional entropies (conditional on a right response and conditional on an incorrect response) using (3).
3. Weight the conditional entropies by their probabilities using (4).

A variant of this approach is relative entropy, which is also called the Kullback-Leibler (K-L: 1951) information measure and information divergence. Chang and Ying (1996), Eggen (1999), and Lin and Spray (2000) have favorably evaluated K-L information as an adaptive testing item selection strategy.

The reader should note that after administering the most informative items, the expected entropy for all the remaining items could be greater than $H(S)$ and result in a loss of information. That is, the classification probabilities would be expected to become less peaked. One may want to stop administering items when there are no items left in the pool that are expected to result in information gain.

**Sample computations**

To illustrate the model and the Information Gain approach, assume three questions have been calibrated against an existing group of masters and non-masters with the item parameters shown in Table 1. Further assume that 68% of the calibration sample participants were masters.

**Table 1: Conditional probabilities of a correct response, $P(z_i=1 \mid m_k)$**

|  | Item 1 | Item 2 | Item 3 |
|---|---|---|---|
| Masters ($m_1$) | 0.8 | 0.9 | 0.6 |
| Non-masters ($m_2$) | 0.3 | 0.4 | 0.2 |

If an individual has a response vector $\mathbf{z} = [1,1,0]$ and the prior probability of being a master is 0.68, then, by formula (2), the probability of their being a master is $(0.8 * 0.9 * (1.0 - 0.6)) * 0.68 = 0.196$ and the probability of their being a non-master is $(0.3 * 0.4 * (1.0 - 0.2)) * 0.32 = 0.031$. This individual is most likely a master and could be classified as such. Rather than use the odd probabilities of 0.196 and 0.031, we can normalize the probabilities. The probability of being a master given $\mathbf{z}$, the group priors, and the item parameters is $0.196 / (0.196 + 0.031) = 0.86$ and the probability of being a non-master is $(1.0 - 0.86) = 0.031/(0.196 + 0.031) = 0.14$.

To illustrate the Information Gain approach to adaptive testing, again assume the conditional probabilities in Table 1, and that the prior probability of being a master is 0.68. Further assume the individual responded correctly to question 1. The task then, is to determine whether question 2 or 3 is likely to provide more information and hence should be administered next. Calculations are shown in Table 2.

Based on the response to the first question, the probability of this examinee being a master is $0.8 * 0.68/(0.8 * 0.68 + 0.3 * 0.32) = 0.85$ and the conditional probability of being a non-master is 0.15. Given the 0.85 and 0.15, the probability of responding correctly to the unadministered question 2 is $0.85 * 0.9 + 0.15 * 0.4 = 0.825$ and the probability of responding correctly to question 3 based on the available information is 0.54.

Applying formula (3) to the above for question 2, the current entropy is $-0.85 * \log_2(0.85) + -0.15 * \log_2 (0.15) = 0.61$. The item selection task, then, is to identify which question, if administered next, is expected to reduce entropy the furthest.

If the examinee does indeed respond correctly to question 2 then the non-normalized probability of being a master and non-master are $(0.8 * 0.9) * 0.68 = 0.49$ and $(0.3 * 0.4) * 0.32 = 0.04$, respectively. Normalized, these probabilities are 0.93 and 0.07, respectively. The entropy if both questions 1 and 2 are right is $-0.93 * \log_2(0.93) + -0.07 * \log_2(0.07) = 0.38$. If the response to question 2 is incorrect then the normalized probabilities of being a master and non-master are 0.49 and 0.51, respectively and the entropy would be 0.99. Considering the probability of responding correctly to question 2 is 0.825, the expected entropy for question 2 given a correct response to question 1 is $0.825 * 0.38 + 0.175 * 0.99 = 0.48$ and the information gain is $0.61 - 0.48 = 0.13$. For question 3, the expected entropy is 0.55 and the expected information gain, $0.61 - 0.55 = 0.06$, is less than that of question 2. Thus question 2 is expected to provide more information and would be selected next.

**Table 2: Computation of expected information gain for items 2 and 3**

| | Response ($z_i$) | Posterior classification probabilities | Conditional entropy | $P(z_i)$ | $H(S_i)$ | Expected Info Gain |
|---|---|---|---|---|---|---|
| **Item 2** | Right | $P(m_1) = 0.93$ | 0.38 | 0.825 | | |
| | | $P(m_2) = 0.07$ | | | 0.48 | 0.13 |
| | Wrong | $P(m_1) = 0.49$ | 0.99 | 0.175 | | |
| | | $P(m_2) = 0.51$ | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| **Item 3** | Right | $P(m_1) = 0.94$ | 0.31 | 0.54 | | |
| | | $P(m_2) = 0.06$ | | | 0.55 | 0.06 |
| | Wrong | $P(m_1) = 0.73$ | 0.83 | 0.46 | | |
| | | $P(m_2) = 0.27$ | | | | |

## Method

The first objective of this research was to evaluate the classification accuracies of a decision-theory adaptive testing approach relative to the classification accuracy of an IRT adaptive testing. This was addressed using simulated datasets based on actual item parameters and actual thetas. In order to replicate a realistic scenario, content and exposure controls were added to both item selection algorithms.

Examinees were simulated by randomly drawing 5,000 ability estimates from a collection of Graduate Management Admission Test (GMAT) quantitative scores. These theta estimates from the GMAT exam were treated as true scores, and each examinee was classified into one of four groups, corresponding to the 0-40th, 40-60th, 60-80th, and 80th-100th percentiles, based on these theta scores. These classifications were treated as the true group classification for the examinees.

IRT item parameters were based on samples of items from an operational GMAT pool. For this part of the study, the *a priori* conditional group probabilities of a correct response for measurement decision theory were computed as the area of the IRT curve within each score band weighted by the proportion of examinees at each theta level within the interval.

The IRT adaptive testing approach started with a test question of about median difficulty. Item responses were simulated using Birnbaum's (1968) three-parameter IRT model. Using the true theta score, the examinee's probability of a correct response was compared to a random number between 0 and 1 and coded as either responding correctly or incorrectly.

After each question, ability was estimated to the nearest of 13 quadrature points using maximum likelihood estimation. Rather than having wild swings in ability estimates and item difficulties, a maximum step size of +/- 3 quadrature points was used for the all-right or all-wrong response vectors. The next item to be administered was randomly selected from the five unadministered items with the right content whose difficulties were closest to the interim ability estimate. The final ability estimate was computed as the maximum likelihood point estimate and then compared to the cut scores to determine the final group classification. The random selection provided exposure control; a bin-structure-based item selection algorithm provided content balancing. Accuracy was defined as the proportion of correct classifications

The MDT adaptive testing approach used Information Gain as previously outlined. Testing started with a randomly selected item from the first bin with near median difficulty. As with the IRT simulation, responses were based on the three-parameter model using the true theta scores. After each question was administered and scored, the interim group classification likelihoods were

computed using uniform priors. The next item to be administered was randomly chosen from the five unadministered items with the highest expected information gain. The item exposure and content balance were implemented in the same way as the IRT CAT approach. The final group classification was the group with the maximum likelihood.

The item bank size was varied, using pools of 200, 300 and 600 questions. While both conditions utilized adaptive methods of item selection, the termination criterion was a fixed length test. Test length was also varied, using lengths of 10, 15, 20, 25, and 30 questions.

Key statistics for this dataset are shown in Table 3.

**Table 3: Key statistics for the IRT and MDT adaptive testing comparison**

| | |
|---|---|
| Mean $\theta$ | 0.6760 |
| SD $\theta$ | 1.2381 |
| Cut 1 | 0.3645 |
| Cut 2 | 0.9975 |
| Cut 3 | 1.7241 |

The second objective was to evaluate classification accuracy as a function of calibration sample size. IRT can require 1,000 or more examinees in order to obtain quality item parameters. Assuming random selection, how many examinees are needed to obtain satisfactory MDT item parameter estimates?

A second analysis was conducted for the MDT approach using calibration sample sizes of 100, 200, 500, 1,000, and 2,000 examinees to obtain the *a priori* conditional probabilities of a correct response for examinees in each of the four groups. The 100 and 200 sample size conditions represent minimal sample sizes that might be used by smaller testing programs. Because a calibration sample size of 1,000 to 2,000 is needed to make fine distinctions along three parameters, it was felt these samples would yield close to population values for the simpler MDT model.

Examinee samples were again drawn from GMAT thetas and responses were again generated by comparing a random value to the response probabilities using the three-parameter model. Examinees were grouped into their true classification and the proportions of examinees within each group that responded correctly were used as the a priori conditional group probabilities. In order to prevent individual questions from dominating the analysis, minimum and maximum conditional probabilities of 0.02 and 0.98 were used. The MDT Calibrate software package (Rudner, 2009) was used. Only one bank of a random sample of 200 questions was used in this second analysis.

Once the item bank was calibrated, the examinations were adaptively administered using the information gain approach. There was no overlap between the calibration samples and the simulees that were administered these MDT CAT examinations.

<p style="text-align:center"><strong>Results</strong></p>

## Adaptive Testing Accuracy

Classification accuracy of IRT and MDT adaptive testing as a function of test length and pool size are shown in Table 4. As expected, as test length increases so does classification accuracy for both the IRT and the MDT approach. Pool size made little difference. When the task is simply to classify, it appears that large pools are not needed. There were also minimal differences between MDT and IRT. The simpler MDT model was as accurate as the more complicated IRT model in terms of accurately classifying examinees into one of four groups.

**Table 4: Classification accuracies of IRT and MDT adaptive testing as a function of test length and pool size**

| Pool size | Model | Test length | | | | |
|---|---|---|---|---|---|---|
| | | 10 | 15 | 20 | 25 | 30 |
| 600 | IRT | .657 | .705 | .736 | .766 | .777 |
| | MDT | .709 | .746 | .775 | .792 | .804 |
| 300 | IRT | .642 | .700 | .731 | .760 | .783 |
| | MDT | .694 | .732 | .754 | .776 | .787 |
| 200 | IRT | .652 | .708 | .740 | .756 | .783 |
| | MDT | .675 | .722 | .742 | .753 | .779 |

Table 5 and Figure 1 show MDT classification accuracy as a function of test length and calibration sample size. It is readily apparent that for any given test length, there is little difference in accuracy between an item bank calibrated on 100 examinees and one calibrated on 2,000. While accuracy does increase with sample size, the increases are small. Test length is a far more important predictor of accuracy.

**Table 5: Proportion of examinees classified correctly as a function of test length and calibration sample size**

| Calibration size | Test length | | | | |
|---|---|---|---|---|---|
| | 10 | 15 | 20 | 25 | 30 |
| 100 | .646 | .688 | .703 | .721 | .741 |
| 200 | .647 | .695 | .723 | .746 | .767 |

| | | | | | |
|---|---|---|---|---|---|
| 500 | .673 | .722 | .743 | .762 | .775 |
| 1,000 | .667 | .716 | .734 | .750 | .764 |
| 2,000 | .674 | .723 | .745 | .770 | .783 |

## Classification accuracy as a function of test length and calibration sample size



Calibration sample size: 100, 200, 500, 1000, 2000
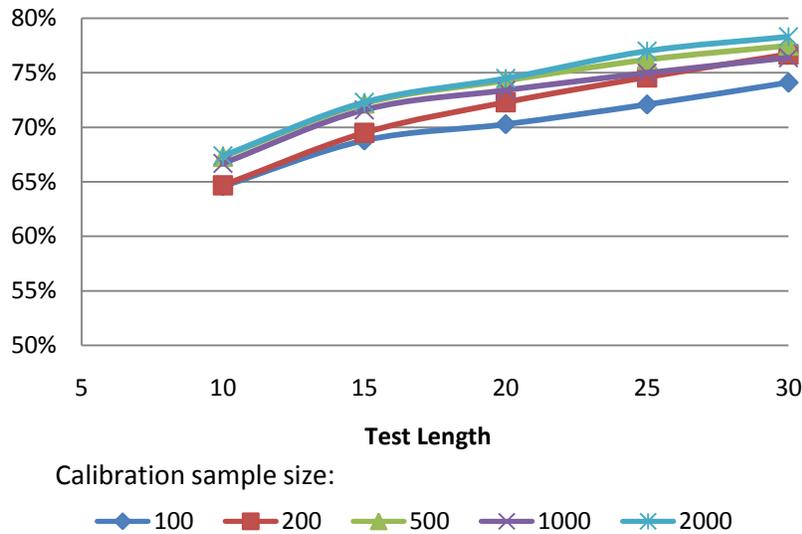
## Discussion

This research shows that the simpler measurement decision theory model can be as effective as the popular three-parameter IRT model in terms of correctly classifying examinees into one of four groups. Using real item parameters and real theta scores, the IRT and MDT CAT algorithms provide for extremely similar percentages of correct classifications. A second analysis shows that calibration samples sizes as small as 100 can yield high classification accuracies in MDT CAT, although we will admit that the perfect examinee classifications in the second analysis may have yielded artificially accurate item calibrations.

The results suggest that MDT is clearly a simple yet powerful and widely applicable model. The advantages of this model are many—the model yields accurate mastery state classifications, can incorporate a small item pool, is simple to implement, requires little pre-testing, is applicable to criterion referenced tests, can be used in diagnostic testing, can be adapted to yield classifications on multiple skills, can employ sequential testing and a sequential decision rule, and should be easy to explain to non-statisticians.

The model can be used as the routing mechanism for intelligent tutoring systems. Items could be piloted with a few examinees to vastly improve end-of-unit examinations. Certification examinations could be created for specialized occupations with a limited number of practitioners available for item calibration. Short tests could be prepared for physicians or teachers to help make tentative decisions. A small collection of items from one test could be embedded in another test to yield meaningful cross-regional information.

While an attractive feature of the model is that it can work with small calibration samples, the use of such small samples can be very problematic. Even if the small sample is representative of the population, the *a priori* item parameters may not be adequately calibrated. With 100 examinees and four groups, an average cell size of 25 people is used to estimate population parameters. The resultant parameters may not be very stable. Coupling poor item parameters with a very short test can be expected to lead to substantial classification error.

A key question not addressed here is the local independence assumption. We naively assumed that the responses to a given item are unaffected by responses to other items. While the local independence is often ignored in measurement and one might expect only minor violations, its role in measurement decision theory is not fully understood. The topic has been investigated in the text classification literature. Despite very noticeable and very serious violations, naive Bayes classifiers perform quite well. Domingos and Pazzani (1997) show that strong attribute dependencies may inflate the classification probabilities while having little effect on the resultant classifications. They argue that naive Bayes classifiers have broad applicability.

One can view MDT as a crude IRT model that uses a step function rather than a continuous function. Indeed MDT parameters in the first investigation used IRT probabilities weighted by frequency. Investigating the likelihood for a limited number of classifications is the same mathematical concept as examining the likelihood of each value on the theta continuum. Viewed this way, one must question whether the use of observed proportions as the MDT a priori item parameters is as good as using an underlying population model such as in IRT. There is a logical disconnect in that regard.

Another issue with the MDT model is that, at present, there is no accepted way to assess reliability within the model. There is statistical literature on consistency and asymptotic normality of maximum likelihood estimates that might prove helpful.

This paper only examined fixed length tests. Thompson (2007) discusses variable length computerized classification testing where the goal is to reduce classification error while using as few questions as possible. Testing continues until the probability of new questions changing the most likely classification category is low. Wald's (1947) sequential probability ratio testing (SPRT), perhaps the foremost application of decision theory, has been used by numerous researchers (e.g., Lin & Spray, 2000; Parshall, Spray, Kalohn, & Davey, 2006; Reckase, 1983) to define termination rules. An interesting line of research would be to apply SPRT to a MDT adapted test. Rudner (2002) reports one such study.

The research questions to spur future studies are numerous. How can the model be extended to multiple rather than dichotomous item response categories? How can bias be detected? How effective are alternative adaptive testing and sequential decision rules? Can the model be effectively extended to 30 or more categories and provide a rank ordering of examinees? How can we make good use of the fact that the data is ordinal? Are there new item analysis procedures that can improve measurement decision theory tests? How can the model be best applied to criterion-referenced tests assessing multiple skills, each with a few items? Why are minimum cost and information gain so similar? How can different cost structures be effectively employed? How can items from one test be used in another? How does one equate such tests?

**Note**s

1. An earlier version of this paper was presented as a keynote address at the International Association on Computer Adaptive Testing in Arnhem, Netherlands, June 2010.

2. The views and opinions expressed in this paper are those of the authors. No endorsement by any organization should be inferred or implied.

3. An interactive tutorial is available online at http://pareonline.net/sup/mdt/ . The tutorial allows you to vary the results of the a priori parameters, the examinee's response pattern, and the cost structure. Various rules for classifying an examinee and sequencing items are then presented along with the underlying calculations.

4. Software for generating, calibrating, and scoring measurement decision theory data is available at http://pareonline.net/sup/mdt/MDTToolsSetup.exe . Updated April 2010, this is version .895. No support is provided.

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